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Abstract

This paper proposes a model for evaluating long-term measures to reduce congestion at airports in the National Airspace System (NAS). This model is constructed with the goal of assessing the global impacts of congestion management strategies, specifically slot restrictions. We develop the Minimax Node Throughput Problem (MINNTHRU), a multicommodity network flow model that provides insight into air traffic patterns when one minimizes the worst-case operation across all airports in a given network. MINNTHRU is thus formulated as a model where congestion arises from network topology. It reflects not market-driven airline objectives, but those of a regulatory authority seeking a distribution of air traffic beneficial to all airports, in response to congestion management measures. After discussing an algorithm for solving MINNTHRU for moderate-sized (30 nodes) and larger networks, we use this model to study the impacts of slot restrictions on the operation of an entire hub-spoke airport network. For both a small example network and a medium-sized network based on 30 airports in the NAS, we use MINNTHRU to demonstrate that increasing the severity of slot restrictions increases the traffic around unconstrained hub airports as well as the worst-case level of operation over all airports.

1 Introduction

In 2007, congestion at airports in the NAS contributed substantially to the 281.4 billion hours of delay experienced by domestic U.S. air passengers [33], and the Federal Aviation Administration (FAA) projects that the number of congested airports within the NAS will only grow [37]. Long-term measures, such as slot restrictions, have been employed at airports to reduce or prevent congestion in advance of operation, but their use has not been widespread nor has there been a consensus about their impacts. This paper proposes a network flow model for assessing the NAS-wide effects of local airport-level decisions to address congestion, specifically the use of slot restrictions. For a given airport network representing a day or other multi-hour period of operation, our model determines the flow of aircraft associated with the minimum worst-case level of operation over all airports. That is, the output of the model describes a routing of aircraft throughout the given network minimizing the maximum amount of traffic at any airport. Using this model, we illustrate the impacts of slot restrictions on small and moderately-sized airport networks with hub-spoke topologies.

A variety of mechanisms are at the disposal of the Federal Aviation Administration (FAA) to handle congestion, both in real-time and for long-term planning purposes. Air Traffic Flow Management (ATFM) encompasses the methods employed by the Federal Aviation Administration (FAA), in collaboration with airspace users (e.g., airlines), for identifying and mitigating short-term congestion within the NAS and particularly at airports [22]. Frequently used and studied ATFM procedures include flight rerouting [4, 10] – the direction of aircraft away from congested portions of airspace – and ground delay programs [5, 39] – the assignment of delayed departure times to flights destined for a congested airport. At a longer time scale, demand management, a set of measures used to constrain access to airports, is a key instrument for controlling congestion and resulting delays far in advance of any scheduled flights. The primary forms of demand management are congestion pricing [30, 31] and slot restrictions [3, 29]. Congestion pricing involves no limit on aircraft takeoffs and landings per unit time instead restricting capacity by adjusting typical aircraft weight-based landing fees [18]. Slot restrictions consist of setting an explicit upper bound on the number of movements per unit time at an airport and allocating a limited number of slots (time intervals for an arrival/departure for a set of dates) among airlines and other aircraft operators.

In the United States, slot restrictions have been used primarily at a select few of the

busiest airports in the NAS, initially using the FAA’s High Density Rules (HDR) at New York Kennedy, New York LaGuardia, Newark, Washington (now Reagan) National and Chicago-O’Hare airports, transitioning through several formats into present-day restrictions still practiced at these airports [18]. Historical experience with slot restrictions at some of these airports has not all been positive (see e.g., [15,16]), but the most recent form of slot restrictions at the New York City metroplex airports (Kennedy, LaGuardia and Newark) has been observed [23] to reduce the frequency and length of delays at those airports.

However, slot restrictions only address local effects. Due to heightened NAS activity, flight delays at the local level have significant global impact on air transportation operations. Investigation of these global effects has been undertaken. For example, Beatty et al. [7] develop the concept of Delay Multiplier to quantify how the initial delay of an individual flight propagates over an airline’s entire operating schedule. After the enactment of the Wendell H. Ford Aviation Investment and Reform Act for the 21st Century (AIR-21) [38], many exemptions to existing slot restrictions at New York’s LaGuardia Airport (LGA) were granted, increasing delays experienced by flights into and out of LGA and resulting in days of NAS operation where “some 376 flights traveling to 73 airports experienced flight delays because their aircraft passed through LaGuardia at least once” [2].

A number of system-level approaches to studying air traffic and congestion management issues in the NAS have been proposed. For instance, Sun and Bayen [35] have formulated a set of models for estimating sector-level capacities and travel times and evaluating ATFM strategies at an aggregate level. Network-theoretic analyses, such as those in [11] and [17], have provided insight into the roles of airports and the influence of airline trends on the structure of the U.S. airport network. A model of the U.S. airport network by Donohue [19] has been used to estimate the network’s capacity and identify metrics affecting system capacity and delay.

This paper contributes to this area of research by proposing a model for the NAS that provides insight into network-wide operational properties of the NAS and evaluates the global impacts of airport-level policies, such as slot restrictions. We consider a general network optimization formulation, the Minimax Node Throughput Problem (MINNTHRU), a variation of standard multicommodity bottleneck flow problems. The objective is to minimize the worst-case amount of flow into or out of any single node in a given network. In air transportation applications, this model, to be presented in Section 2, determines the flow of aircraft associated with the minimum worst-case operation across all airports over the course of several hours to a day for a given airport network. Solutions to MINNTHRU may provide insight into airports with over/underutilized capacity as well as how a slot restriction at a particular congested airport affects air traffic throughout the network without overloading any individual airport in a given network. The MINNTHRU formulation presented here is meant to be a first step in an investigation of global phenomena in air transport networks, reflecting the perspective of the FAA and other regulatory authorities. The primary objective of the model is to identify congested airports and to understand an individual effect of constraining traffic around these airports – namely, how overall air traffic shifts under a specified network topology, while keeping the maximum traffic through any airport as small as possible. The current formulation does not incorporate the actions of airlines and other airspace users, among model assumptions. In Section 3 we discuss our assumptions and the extensions to MINNTHRU that can be made to address them.

Using the formulation in Section 2, we demonstrate that the dimension of MINNTHRU grows at a cubic rate, making even moderately-sized instances of MINNTHRU (for networks of 20-30 nodes) intractable for standard optimization packages and necessitating the development of alternative algorithms. We describe such an algorithm for approximately solving MINNTHRU in Section 4. The algorithm employs a bisection routine to solve a finite number of feasibility problems using a Lagrangian Relaxation-based decomposition

method. Having developed an algorithm to solve moderately-sized and larger instances of MINNTHRU, we present in Section 5 an application of MINNTHRU to the analysis of the effects of slot restrictions in hub-spoke airport networks. We first use a small, six-node network to allow for a detailed and instructive illustration of how MINNTHRU can provide insights into the effects of a slot restriction on a hub airport. Then, using a 30-node network whose nodes correspond to NAS airports, we study how slot restrictions on a hub affect operations at unconstrained airports and the minimum worst-case level of performance over all airports. For both small and medium-sized hub-spoke network examples, we demonstrate that increasing the severity of slot restrictions at hub airports results in a similar increase in both the worst-case level of operation throughout the network and in the flow of aircraft through unconstrained hub airports. We revisit the results of this application and discuss their implications for airport operations and airline schedules in Section 6, while also revisiting the extensions of the basic MINNTHRU model in Section 2 and their potential contributions to the management of congestion in the NAS.

2 The Minimax Node Throughput Problem (MINNTHRU)

We now offer a basic formulation for MINNTHRU, then describe the formulation in an air transportation context and its applications to managing airspace/airport capacity. Throughout this section, we use the following notation:

- N : the set of nodes
 - SC : a subset of N , the set of nodes with a flow constraint
- A : the set of arcs, where an arc is the ordered pair (i, j) such that $i, j \in N$ and $i \neq j$
- K : the set of commodities, where a commodity $k \in K$ is defined by the triplet (s_k, t_k, d_k) with
 - s_k : the source node of commodity k
 - t_k : the destination node of commodity k
 - d_k : the demand vector associated with commodity k
- x_{ijk} : the amount of flow of commodity k on arc (i, j)
- x_k : for each commodity k , the vector of all (i, j) arc flows x_{ijk}
- u_i : the upper bound on of flow through node $i \in SC$
- \mathcal{N} : the node-arc incidence matrix, such that $\mathcal{N}_{ij}=+1$ if arc (i, j) is directed away from node i , $\mathcal{N}_{ij} = -1$ if arc (i, j) is directed towards node i and $\mathcal{N}_{ij} = 0$ otherwise
- $|W|$: for any set W , the cardinality of W .

2.1 The Basic Formulation

We represent a multicommodity flow network by the simple, connected and directed graph $G = (N, A)$, over which $|K|$ commodities are to be transported. Each commodity $k = (s_k, t_k, d_k) \in K$ is associated with an ordered pair of nodes (s_k, t_k) , requiring a quantity of flow to be transported originating at s_k and ending at t_k . Associated with each commodity is the demand vector d_k whose s_k th entry is non-negative, t_k th entry is the additive inverse of

the s_k th entry and remaining entries are zero, specifying the exact quantity to be transported starting from s_k and ending at t_k . We now formulate MINNTHRU as

$$\min_{x,z} \quad z \quad (1)$$

$$s.t. \quad \sum_{k \in K} \sum_{j \in O(i)} x_{ijk} \leq z, \quad \forall i \in N \setminus SC \quad (2)$$

$$\sum_{k \in K} \sum_{j \in O(i)} x_{ijk} \leq u_i, \quad \forall i \in SC \quad (3)$$

$$\mathcal{N}x_k = d_k, \quad k \in K \quad (4)$$

$$x_{ijk} \geq 0, \quad \forall (i, j) \in A, \forall k \in K, \quad (5)$$

where for each $i \in N$, $O(i) = \{m \in N | (i, m) \in A\}$ and z denotes the feasible flow capacity on flow on all nodes. From the above formulation, the optimal solution to MINNTHRU, (z^*, x^*) , provides, for a given network, the worst-case flow through any unconstrained node and the set of flows associated with this worst-case level.

Adapting MINNTHRU for an air transportation application, N corresponds to a set of airports within the NAS, SC the set of airports within the NAS subject to slot restrictions, A to all unidirectional one-stop connections that exist between each airport in N , and each of the $|K|$ commodities to each unique origin-destination route pair possible in the network. The demand associated with each commodity $k = 1, \dots, |K|$ corresponds to the number of completed itineraries flown by an individual aircraft between for the O-D pair (s_k, t_k) during a given time window and flows are the quantity of flights between each distinct node pair $i, j \in N$. We can view the optimal objective function value of MINNTHRU as the worst-case number of aircraft movements any airport can handle for the period of operation represented by the network, a local measure of airport congestion.

A more systemic perspective on potential congestion in the NAS comes from the full solution of MINNTHRU, (z^*, x^*) . The full solution represents the flow of aircraft in a given network associated with the worst-case level of operation, allowing for computation of relative levels of congestion for each airport with respect to the most-congested one. From the full solution of this model, we can identify theoretically feasible levels of operation for any given network topology, representing all available flight paths and demands on aircraft, and compare them to actual traffic patterns, perhaps allowing for a determination of the extent to which the flight schedule itself explains certain congestion phenomena. Along the same line of investigation, we can use existing measures of airport capacity, such as the FAA's own capacity benchmarks [21], in comparison to a MINNTHRU solution to identify portions of the NAS where there may be over/underutilized capacity to focus upon directing traffic towards/away.

Applying the MINNTHRU formulation to more dynamic purposes, we can compare the solution of less-constrained MINNTHRU instances (e.g., where there are no slot-restricted airports and so SC is empty) to instances where SC is non-empty, allowing for a very simple characterization of the effects of slot restrictions at a few select airports on the overall flow of traffic through the NAS. A deeper analysis of these effects can be generated by solving a number of MINNTHRU instances with the same set of slot-restricted airports SC and varying the movement capacities in small (e.g., unit) increments to compute marginal traffic flow estimates. Applying further the approach of Fan and Odoni [20] to calculating marginal delays and delay costs, we can provide baseline estimates of the "value" of capacity at congested airports.

3 MINNTHRU Model Assumptions

As presented in the previous section, the MINNTHRU model can provide a global perspective on the effects of congestion around airports in the NAS. The current model’s intent is to give high-level insight into these effects to regulatory authorities. Thus, the model uses the flow of aircraft as the primary effect of airport-level congestion and strategic measures to alleviate congestion. We have not considered the actions of airlines and other airspace users and how they are affected by congestion and policies, such as slot restrictions. By not fully incorporating these behaviors, the present MINNTHRU model assumes several simplifications of airline behaviors.

First, we have assumed a single airline with the intent of presenting MINNTHRU as a “system” model. Since the model emphasizes analyzing congestion management on a global scale, and thus places importance on the total level of traffic around airports, this assumption is reasonable at this stage of investigation. It may, however, lead to practically infeasible solutions which do not account for specific airline hubs, roles of particular airlines (e.g., regional vs. major air carrier), etc. We can add authenticity to the model’s results by simply having an individual MINNTHRU instance for each airline we consider, then use each airline’s specific network and historical slots at slot-restricted airports, and feed the results of these MINNTHRU subproblems into a “master” MINNTHRU problem. An issue that remains is the economics of each airline moving its aircraft in response to other airlines and slot restrictions or other congestion management measure. This is an important issue that will be addressed in future work.

The next assumption is that the operation of the one aggregate airline is simplified to quite an extreme. In particular, revenues and costs are not incorporated in the model, potentially leading to nonsensical aircraft routings (e.g., completing an itinerary whose origin and destination are on the same coast by routing an aircraft to the opposite coast and back) and other routing decisions leading to lost revenues in actual operation. We have not considered many other types of decisions made by airlines, such as the choice of aircraft and pricing of flight itineraries. Again, the solutions returned by MINNTHRU may be unprofitable after factoring in these decisions. A potential extension would require: (1) modifying the minimax objective function from a strictly flow-based objective to a penalty function accounting for prices, flight costs (based on distance and choice of aircraft), and delay costs based on the congestion around airports and (2) expanding the commodity space to differentiate between flights by different aircraft types.

Finally, we have not considered the relationship between passenger demand and the management of airport congestion. Still, the demands on aircraft are driven in part by travelers with a desire to fly, so how passenger demand is affected by congestion and its management and the resulting responses of airlines cannot be ignored completely. An analysis of the role passenger demand plays in congestion management might require a derivation of a relationship between an airline’s passenger and aircraft demands, in conjunction with a model of the economic decisions faced by airlines, and then solving MINNTHRU for a number of instances with different demand levels.

Assuming a single airline and not incorporating a notion of passenger demand in MINNTHRU do not present severe limitations to our model since it is mainly a model of the air transport system as a whole, and focuses upon comparing total levels of aircraft operations at airports. It is clear, however, that the various assumptions about the behavior of the single airline limit our model the most, and that incorporating economic concepts would result in MINNTHRU solutions less likely to be unprofitable or not sensible in real airline operations. Our model still has value in that we have proposed it as a preliminary step catered to the interests of regulatory authorities aiming to understand *some* of the global effects of its congestion management policies. In this sense, this version of MINNTHRU

provides an idealized view of the effects of these policies considering only the flow of aircraft, allowing for the characterization of lower bounds on worst-case traffic and marginal changes in flow resulting from slot restrictions.

4 An Algorithm to Solve MINNTHRU

In the formulation given by (1)-(5), we have assumed that the commodities $1, \dots, |K|$ correspond to each distinct and ordered pair of nodes, so it follows that MINNTHRU is a problem with $O(|N|^3)$ constraints in $O(|N|^2|A|)$ variables. For networks where $|N|$ and $|A|$ are large, MINNTHRU is intractable. For example, the linear programming routine `linprog` in MATLAB 7.8's Optimization Toolbox [28] can solve a MINNTHRU instance of no larger than 15 nodes and approximately 80 arcs. We now describe an algorithm for solving larger instances of MINNTHRU, using the minimax structure of the formulation given in (1)-(5). Our algorithm consists of a main loop in which we conduct a binary search over the interval $[0, |K|]$ for candidate values of z^* , the optimal objective function value of MINNTHRU, solving a feasible flow problem for each candidate value z , until the search interval is sufficiently reduced. Within the main loop, we must solve a feasible flow problem for each z that is evaluated. These feasible flow problems are each a large multicommodity flow problem we can solve using well-known decomposition methods based on Lagrangian relaxation [1] and subgradient optimization techniques [34].

4.1 Description of the Algorithm

Let us define the parameters c_{lo} and c_{hi} , which are, respectively, lower and upper endpoints of the search interval for potential values of z^* . Let $\delta > 0$ be the tolerance for the difference between c_{lo} and c_{hi} , thus serving as a termination criterion to the main loop of the algorithm. We initialize $c_{lo} = 0$ and $c_{hi} = |K|$, since there can be no flow into/out of any node greater than the total number of commodities. The main loop is as follows:

1. Set $z = (c_{hi} + c_{lo})/2$ and execute the procedure $DECOMP(z)$.
2. If the result is FEASIBLE set $c_{hi} = z$, if INFEASIBLE, set $c_{lo} = z$.
3. If $(c_{hi} - c_{lo})/z < \delta$, terminate. Otherwise return to step 1.

We use the procedure $DECOMP(z)$ to determine whether a particular value of z is feasible by specifying for each z , the following feasible flow problem associated with the MINNTHRU formulation from (1)-(5):

$$\min_x p \tag{6}$$

$$s.t. \quad \sum_{k \in K} \sum_{j \in O(i)} x_{ijk} \leq z, \quad \forall i \in N \setminus SC \tag{7}$$

$$\sum_{k \in K} \sum_{j \in O(i)} x_{ijk} \leq u_i, \quad \forall i \in SC \tag{8}$$

$$\mathcal{N}x_k = d_k, \quad \forall k \in K \tag{9}$$

$$x_{ijk} \geq 0, \quad \forall (i, j) \in A, \forall k \in K, \tag{10}$$

where $x = \{x_k\}$. This problem amounts to finding an x that satisfies the system of inequalities defined by (7)-(10). Denoting the optimal objective function value of this problem by

$p^*, p^* = 0$ if (7)-(10) is feasible and $p^* = \infty$ if not.

Forming the Lagrangian subproblem for the feasible flow problem (6)-(10), we have:

$$L_z(v, w) = \inf_x \sum_{i \in N \setminus SC} v_i \left(\sum_{k \in K} \sum_{j \in O(i)} x_{ijk} - z \right) + \sum_{l \in SC} w_l \left(\sum_{k \in K} \sum_{j \in O(l)} x_{ijk} - u_l \right) \quad (11)$$

$$s.t. \quad \mathcal{N}x_k = d_k, \quad \forall k \in K \quad (12)$$

$$x_{ijk} \geq 0, \quad \forall (i, j) \in A, \quad \forall k \in K, \quad (13)$$

where v_i and w_l as the Lagrange multipliers associated with the capacity constraints on node $i \in N \setminus SC$ and $l \in SC$, and $v = \{v_i\}$ and $w = \{w_l\}$ as the vectors of the Lagrange multipliers. We may express this more compactly through the dual function

$$L_z(v, w) = \inf_{x \in X} \sum_{i \in N \setminus SC} v_i \left(\sum_{k \in K} \sum_{j \in O(i)} x_{ijk} - z \right) + \sum_{l \in SC} w_l \left(\sum_{k \in K} \sum_{j \in O(l)} x_{ijk} - u_l \right), \quad (14)$$

where $X \subseteq \mathbb{R}^{|A||K|}$ is the set of all x satisfying the non-negativity and flow conservation constraints (12)-(13). The dual problem associated with (14) is to maximize $L_z(v, w)$ subject to $v \geq 0$ and $w \geq 0$. Denoting d^* to be the optimal value of the dual problem, we observe that we can make d^* as large as possible, depending on what $x \in X$ achieves the infimum for a particular (v, w) in (14). By weak duality, $d^* \leq p^*$, so for any $v \geq 0$ and $w \geq 0$, such that if solving the dual problem results in $L_z(v, w) = \infty$, $p^* = \infty$ and the inequality system (7)-(10) is infeasible. The alternative result, however, is the case where $L_z(v, w) \leq 0$ for any $v \geq 0$ and $w \geq 0$, indicating that (7)-(10) is indeed feasible, in which case $p^* = 0$. It follows that

$$v, w \geq 0, L_z(v, w) > 0 \Leftrightarrow (7)-(10) \text{ is infeasible.} \quad (15)$$

Thus, the procedure *DECOMP*(z) can determine if a particular value of z is infeasible if it can find a v and w such that the left hand side of (15) is satisfied within a sufficiently large number of iterations.

We can now describe the procedure *DECOMP*(z) as follows:

1. *Initialization* Set $q = 1$, $w_i^q > 0$ for all $i \in N \setminus SC$, $v_l^q > 0$ for all $l \in SC$ and define Q , the maximum number of iterations, and $\epsilon > 0$ the feasibility tolerance parameters. Solve $|K|$ single-commodity minimum cost flow subproblems where for each commodity k , arc (i, j) has cost v_i^q if $i \in N \setminus SC$ (cost w_l^q if $l \in SC$) and unit capacity.
2. Let y_{ijk} be the optimal solution to the k th subproblem. If for all $i \in N \setminus SC$, $\sum_{k \in K} \sum_{j \in O(i)} y_{ijk} \leq z$ and for all $l \in SC$, $\sum_{k \in K} \sum_{j \in O(l)} y_{ljk} \leq u_l$, then (6)-(10) is FEASIBLE and terminate. If

$$\sum_{i \in N \setminus SC} v_i^q \left(\sum_{k \in K} \sum_{j \in O(i)} y_{ijk} - z \right) + \sum_{l \in SC} w_l^q \left(\sum_{k \in K} \sum_{j \in O(l)} y_{ljk} - u_l \right) > \epsilon, \quad (16)$$

or if $q > Q$, then (6)-(10) is INFEASIBLE and terminate.

3. Update v_i^q and w_l^q for each $i \in N \setminus SC$ and $l \in SC$, respectively, according to the

subgradient update formulas:

$$v_i^{q+1} = v_i^q + \theta_q \left[\sum_{k \in K} \sum_{j \in O(i)} y_{ijk} - z \right]^+, \quad (17)$$

$$w_l^{q+1} = w_l^q + \theta_q \left[\sum_{k \in K} \sum_{j \in O(l)} y_{ljk} - u_l \right]^+, \quad (18)$$

where $[x]^+$ is the positive part of x and $\theta_q = a/(b + q)$ is a stepsize parameter with positive constants a and b , and set $q = q + 1$.

4. At iteration $q = 2, \dots, Q$, form the residual graph G_k^q under the current flow y_{ijk} and dual prices v_i^q and v_l^q and run the procedure $BFNC(G_k^q)$ to solve the subproblem for each commodity $k = 1, \dots, |K|$ and return to step 2.

The procedure $BFNC(G)$ for any graph $G = (N, A)$ is a version of the Bellman-Ford algorithm [8, 25] with the addition of a procedure for identifying negative-cost directed cycles. After a dual price update, the optimal solution for some of the $|N| \cdot |N - 1|$ commodities may remain optimal while for others it may not. We can identify the commodities whose solutions do not remain optimal by constructing, for each commodity k at iteration q , the residual graph G_k^q resulting from the current flow y_{ijk} and updated dual prices w_i^q and running $BFNC(G_k^q)$ to determine if there is a negative-cost cycle around which we can reroute the flow y_{ijk} at lower cost. If there is one, we augment flow over this cycle to obtain a new solution under dual prices w_i^q , form the resulting new residual graph, and continue running $BFNC(G_k^q)$ until there is no negative-cost cycle. At that point, we keep the solution y_{ijk} and move on to the next commodity. Letting r denote the current Bellman-Ford iteration, and $V_r(i)$ and $F_r(i)$ the distance label and predecessor of node i after Bellman-Ford iteration r , we summarize the procedure $BFNC(G)$ for the case where $G = G_k^q$ for commodity k and iteration q as follows:

1. Run $|N| - 1$ Bellman-Ford iterations on G and record $V_r(i)$ and $F_r(i)$ for each node $i \in N$ for $r = |N| - 1$.
2. Run one additional Bellman-Ford iteration. If $V_{r+1}(i) = V_r(i)$ for all $i \in N$, terminate and return the flow y_{ijk} . Otherwise, move to step 3.
3. Pick any node j such that $F_{r+1}(j) \neq F_r(j)$. Trace back from j and $F_{r+1}(j)$ through the predecessors V_r until a cycle C is identified. Find the capacity M of the cycle and augment M units of flow across C .
4. Update the flow y_{ijk} and the graph G and return to step 1.

4.2 Discussion of the Algorithm

The algorithm described in the previous section combines several existing methods to solve MINNTHRU. Employing a bisection algorithm to solve a series of feasibility problems in order to solve a more complicated optimization problem, as in our algorithm's main loop, is a typical procedure (see e.g., [12], Chapter 4.2) used in solving both quasiconvex and minimax optimization problems. MINNTHRU belongs to this class of problems. It follows simply that the main loop of our algorithm will terminate in $\log |K|/\delta$ iterations, being a binary search over an interval of length $|K|$.

Within the main loop, we employ a Lagrangian relaxation-based, price-directive decomposition scheme that has been employed extensively for standard minimum-cost multicommodity network flow problems as detailed in numerous references, such as [1, 24, 32]. We modify the standard price-directive approach to incorporate node (rather than arc) capacity constraints that are to be relaxed, as well as by using feasibility problems whose optimal values are identically zero as Lagrangian subproblems. Using the procedure *DECOMP*(z), for a given z , we find a sequence of dual prices $(v^1, w^1), \dots, (v^q, w^q), \dots$ which can be shown to converge [26] to the optimal (v^*, w^*) maximizing the dual function $L_z(v, w)$ under the following conditions on the stepsizes θ_q :

$$\theta_q \rightarrow 0, \quad \sum_{q=0}^{\infty} \theta_q = \infty \quad (19)$$

and we have selected one of the more simple forms of a stepsize satisfying these conditions.

It has, however, been noted (e.g., [13]) that the Lagrangian relaxation and subgradient-based decomposition algorithms inspiring *DECOMP*(z) are slow to converge. As part of addressing these issues, we observe that any decomposition breaks MINNTHRU into a set of single-commodity problems which can be solved as shortest-path problems where there are opportunities to reroute flow at arbitrarily lower cost, hence our use of the Bellman-Ford algorithm with negative-cost cycle detection and identification in the graph $G = (N, A)$, *BFNC*(G) in *DECOMP*(z). This application of a shortest-path/negative-cycle detection algorithm to solving multicommodity flow problems has been implemented extensively, see [32] for one example.

Our specification of *BFNC*(G) relies on one of the simpler methods [1, 14, 27] for detecting a negative-cost cycle in a given graph: run the Bellman-Ford algorithm at least $|N|$ iterations and check for a decrease in the distance label $L_r(i)$ in any node $i \in N$ during iteration $r = |N|$. If there is no cycle, Bellman-Ford terminates after at most $|N| - 1$ iterations. With a cycle, all distance labels $L_r(i)$ for each $i \in N$ become arbitrarily negative. [14] We choose to run twice as many Bellman-Ford iterations to let the presence of any negative-cost cycle C run its course, so to speak, before attempting to specifically identify it. The actual procedure for identifying C takes advantage of including the predecessors F_r at each Bellman-Ford iteration r . Then we can backtrack from any node whose distance label continues to update beyond $|N| - 1$, and add all nodes found through F_r until some node is repeated.

5 The Global Effects of Slot Controls: Illustrative Case Studies

In this section, we illustrate the application of MINNTHRU to an investigation of global impacts of local, airport-level slot restrictions through two case studies. In both cases, the input networks to MINNTHRU are hub-spoke networks where spoke-to-spoke travel must utilize a hub, i.e., there are no point-to-point connections. We start with a two-hub, six-node example whose purpose is to show the fundamental differences between unconstrained and constrained (slot-restricted) cases of MINNTHRU, in terms of the decision variables, feasible solutions, and optimal solutions. We then introduce a NAS-like network of 30 nodes, which include five hubs, corresponding to NAS airports. Using this network, we study on a larger scale, how slot restrictions affect the flow of aircraft, particularly the worst-case levels across all airports and the increases in operations on unconstrained airports, especially other hubs.

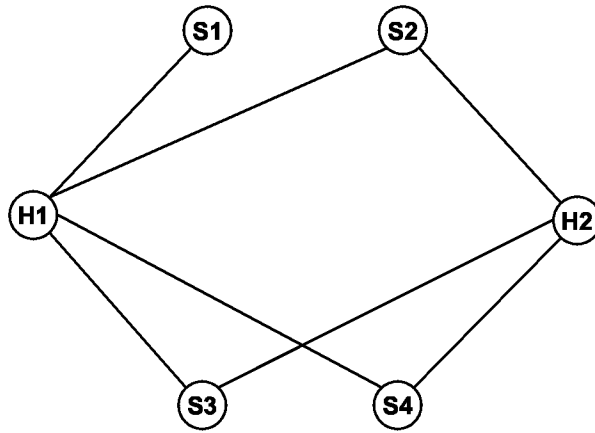


Figure 1. Simple network that illustrates MINNTHRU. H1 and H2 act as hubs connecting spokes S1, S2, S3 and S4. Commodities are associated with pairs of spokes and have unit demand.

5.1 A Small Example

We begin with a simple hub-spoke network depicted in Figure 1, consisting of two hub nodes (H1 and H2), such that traveling between any distinct pair of the four spoke nodes (S1-S4) requires a stopover at either hub. Each edge connecting a pair of nodes in the network corresponds to an arc in both directions between those nodes. Assume also that there is a unit demand for each commodity corresponding to a distinct pair of spoke nodes and zero demand for any other pair of nodes in the network, so that there are twelve commodities in total. Each of these commodities is analogous to a complete routing of an individual aircraft from an origin airport to a destination airport over a relevant service period. That is, there are no aircraft which fly between the two hubs or from any spoke to any hub exclusively.

Now consider finding a MINNTHRU solution for the network in Figure 1, with the given commodities and demands, when there are no flow constraints on any node. As each spoke node serves as an origin for three commodities (where the destinations are each of the remaining spokes), any feasible assignment of flow results in at least three units of flow on arcs outgoing from each spoke node. Since spoke S1 is only connected to hub H1, it is clear that the six commodities, in which S1 is either an origin or destination, must travel through H1. It follows that in any feasible assignment of flow, there will be at least six units of flow on arcs outgoing from H1. The remaining six commodities do not have to be routed exclusively through just H1 or just H2, they may be routed through either or both. However, we can feasibly route all six of these commodities by sending flow exclusively through H2, resulting in at least six units of flow on outgoing arcs from H2. Observing that all commodities with S1 as either origin or destination may be routed exclusively through H1, we arrive at an assignment of flows in which all spoke nodes have three units of flow on outgoing arcs and both hubs have six units of outgoing flow. This assignment is the optimal solution and six units of flow is the unconstrained minimax node throughput, since any alternate routing for any commodity increases the amount of flow on either H1 or H2 and results in greater than six units of flow outgoing from either hub, and we have already established that there cannot be fewer than six units of outgoing flow from all nodes in any

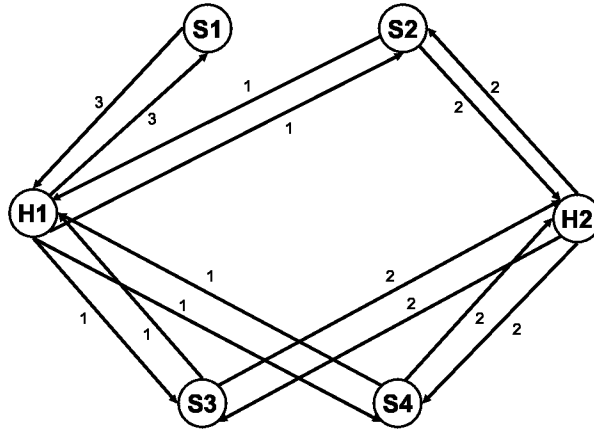


Figure 2. Optimal unconstrained MINNTHRU solution for the network from Figure 1. An arc (i, j) corresponds to a positive flow of aircraft from airport i to airport j . The numbers on any arc represent the units of flow on that arc associated with the optimal solution. We can easily see that all spokes S1 through S4 have three units of outgoing flow and both H1 and H2 have six units of outgoing flow.

feasible assignment of flow. We depict this solution in Figure 2.

Suppose now we implement a slot restriction by constraining outgoing flow on any node in the network depicted in Figure 1. Since each spoke must have at least three units of flow on outgoing arcs and we have shown that we can arrive at a solution where there are exactly three units of flow outgoing from all spoke nodes, no flow constraint on a spoke node which admits a feasible solution (that is, constraining outgoing flow from any spoke to less than or equal to at least three units) will alter the optimal MINNTHRU solution from that of the unconstrained case. Similarly, a constraint on outgoing flow on H1 that admits a feasible solution (constraining outgoing flow to be less than or equal to at least six units) does not change the optimal MINNTHRU solution from that of the unconstrained case, since we have shown that we can arrive at a solution where there are exactly six units of flow outgoing from H1 so that any constraint at least as large does not affect the flow assignment.

The same cannot be said for a constraint on flow outgoing from H2. Denote the upper bound on outgoing flow from H2 by $u_{H2} > 0$. We see that there is no level $0 \leq u \leq 12$ such that we can set $u_{H2} < u$ and make the problem infeasible since no commodities must be routed exclusively through H2. If we set $u_{H2} \geq 6$, then we cannot alter the optimal MINNTHRU solution from that of the unconstrained case since the optimal objective of six units of outgoing flow from H2 in the optimal unconstrained solution remains feasible. However, if we set $u_{H2} < 6$, the optimal unconstrained MINNTHRU solution will change since the six units of outgoing flow from H2 can no longer be accommodated. Since each commodity in which S1 is neither origin nor destination can be routed with H1 alone, at least $6 - u_{H2}$ units of outgoing flow are shifted from H2 to H1, resulting in at most u_{H2} units of outgoing flow from H2 and at least $6 + (6 - u_{H2})$, or $12 - u_{H2}$ units of outgoing flow from H1, with all outgoing flows on spoke nodes remaining at three units. At optimality, the minimum possible flow is rerouted with the outgoing flow from H2 at u_{H2} units and from H1 at $12 - u_{H2}$ units, the latter becoming the minimax node throughput when a constraint of $u_{H2} < 6$ is imposed on outgoing flow from H2. Figure 3 illustrates how the solution

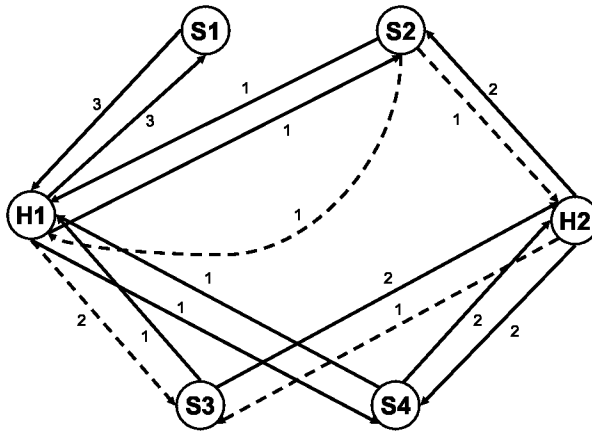


Figure 3. Optimal MINNTHRU solution for the network from Figure 1, when we set the upper bound on outgoing flow from H2 $\mu_{H2} = 5$. To obtain this solution, the commodity associated with O-D pair (2,3) is rerouted through H1. The dashed arcs indicate the changes in the assignment of flows in this constrained optimal solution compared to that of the unconstrained optimal solution. Again we clearly observe that all spokes have three units of outgoing flow, but now H2 has only five units of outgoing flow and H1 has seven units.

changes when we set $u_{H2} = 5$ and arbitrarily reroute the commodity for the O-D pair (2,3) through H1 instead of H2.

5.2 A NAS-like Example

We now consider a larger network (Figure 4) based on the NAS. We choose 30 nodes corresponding to a subset of the 35 FAA Operational Evolution Partnership (OEP) airports considered to be the busiest airports in the United States, and model aggregate operations over these airports for a six-hour block. We exclude OEP airports outside the continental United States and close to other, busier airports. These excluded airports are Chicago Midway (MDW), Fort Lauderdale-Hollywood International (FLL), Honolulu International (HNL), New York John F. Kennedy International (JFK), and Reagan National (DCA). Each line between airports, as in the small network example, indicates that travel is possible in both directions between the two airports. It should be noted that while we selected real airport locations, this example does not represent the actual operations of the airports used.

From the network in Figure 4, we can see five hubs corresponding to Atlanta Hartsfield International (ATL), Chicago O'Hare International (ORD), Dallas-Fort Worth International (DFW), Denver International (DEN), and Minneapolis-St. Paul International (MSP), with the remaining 25 airports acting as spokes. ORD (whose connections are indicated by black lines) is the most connected of these, as it is connected to 20 other airports, followed, in order, by ATL (gray lines) with 16 connections, DEN (blue lines) and DFW (red lines) with 15 connections each and MSP (green lines) with 13 connections. Let us denote these hubs H1 through H5, in ascending order of their degrees so H1 corresponds to MSP, H2 to DFW, H3 to DEN, H4 to ATL and H5 to ORD. For each commodity whose origin and destination nodes are both spokes, there is a unit demand representing one aircraft completing an itinerary between the involved nodes. All remaining commodities have zero demand: that



Figure 4. Example NAS-like network with airports corresponding to 30 of the FAA OEP airports. Connections to hub airports are indicated as follows: H1 (green lines), H2 (red lines), H3 (blue lines), H4 (gray lines) and H5 (black lines). All remaining airports are spokes. Commodities are associated with pairs of spokes and each have unit demand.

meaning aircraft do not fly complete itineraries from hub to spoke (or vice versa) or from hub to hub.

For this example, we will study the global effects of slot restrictions by looking at the following two questions:

- Do slot restrictions induce a rearrangement of the flow of aircraft that results in the most connected, unconstrained hub airport receiving the most flow or the biggest increase in flow?
- Do stricter slot restrictions necessarily increase the minimum worst-case level of flow through any airport (i.e., the minimax node throughput)?

In the small network example, the answer to both questions was affirmative. While this example is clearly more complicated, due to the larger number of airports, hubs and connections, we should expect the answer to the latter question to be similar. With the added complexity it is reasonable to expect that the minimum worst-case level of flow through any airport might not change substantially until the slot restrictions become increasingly strict. However, given the increase in airports, hubs and connections, it is not reasonable to expect that the most connected unconstrained airport will experience either the greatest raw amount or increase in flow of aircraft after a slot restriction. In order to answer these questions, we will use the network depicted in Figure 4 as an input to MINNTHRU, first with no slot restrictions on any airport. After identifying the bottleneck airport and the level of outgoing flow associated with it, we place slot restrictions on the bottleneck airport in decrements of five from the unconstrained minimum worst-case flow and observe the results from using these networks as inputs to MINNTHRU. If there is a tie for the bottleneck airport in the unconstrained case, then the same slot restrictions are applied to no more than two of the airports, since there are only five hubs and these are going to be the airports with the most outgoing flow.

With five hubs and 30 overall airports in our example network, there are too many possible slot restriction combinations to evaluate to definitively answer both of the above questions. In both cases, we focus on the hubs, since – as in the smaller example – in the optimal MINNTHRU solution, the outgoing flow from each spoke is the flow associated with the commodities where the spoke is the origin airport. To address the first question, we will only evaluate the effects of slot restrictions at the bottleneck airports to illustrate these effects. We will take a more systematic approach in applying slot restrictions to evaluate the second question, evaluating results for progressively more restrictive slot restrictions applied one at a time. Without slot restrictions, the optimal solution to MINNTHRU for the network in Figure 4 results in the flow of aircraft through the five hubs as follows: 172 units for H1, 138 units for H2, 172 units for H3, 84 units for H4, and 50 units for H5. We observe that the most connected hub, H5, has the lowest amount of outgoing flow, while two lesser-connected hubs, H1 and H3, have the most flow. While this result may not seem intuitive, recall that the objective of MINNTHRU is minimizing the worst-case flow through any airport and not finding the maximum throughput for any airport.

With a tie between two hubs for having the minimum worst-case flow at 172 units, we put slot restrictions on both H1 and H3, first constraining each node to accommodate no more than 170 units of outgoing flow, then decrementing from that level in units of five. Figure 5 displays the increase in the outgoing flow through the unconstrained hubs H2, H4, and H5 from their unconstrained levels as functions of the capacity reduction on the bottleneck airports H1 and H3 from their unconstrained levels. The initial reduction of capacity on H1 and H3 by just two units induces a substantial increase in flow through both H2 and H4, with further reduction only leading to more flow on H4 and H5. The most significant increase is eventually experienced by H5, the most connected hub in Figure 4, once the

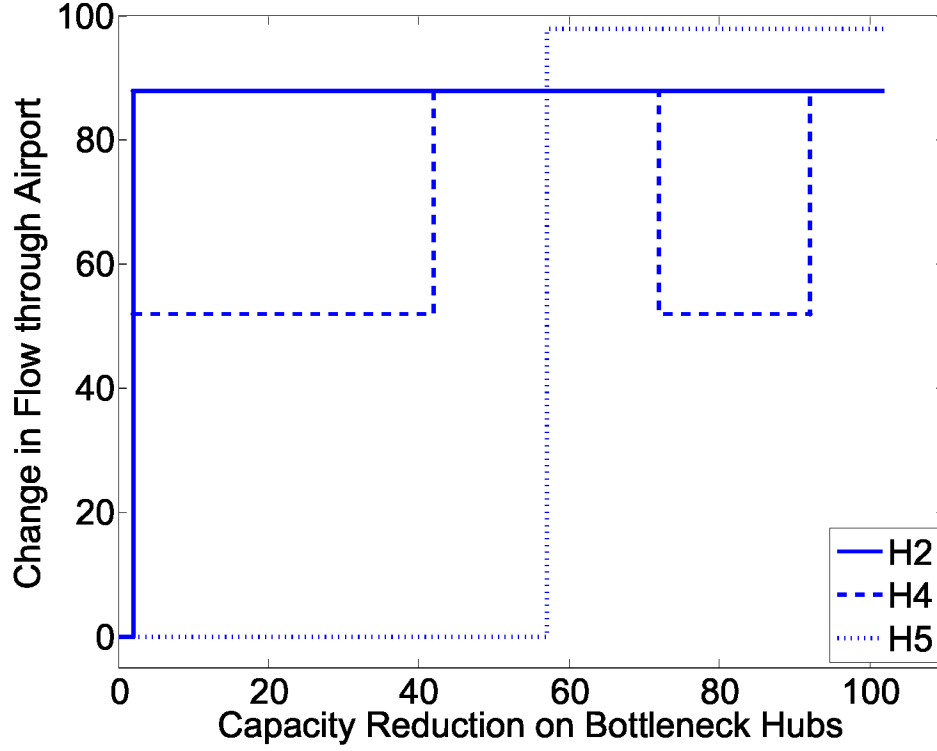
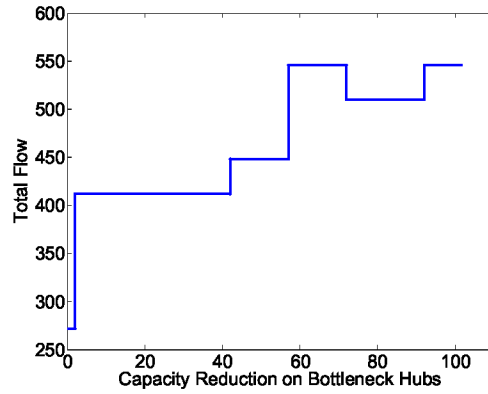


Figure 5: The increase in outgoing flow on each of the three unconstrained hub airports in the network from Figure 4, as a function of the reduction in the capacity of the bottleneck hubs H1 and H3 from their unconstrained level of operation of 172 units each. All spoke airports are unconstrained throughout. Expectedly, the flow through each unconstrained hub increases as the capacities on H1 and H3 are reduced, with the most connected of these hubs, H5, eventually experiencing the largest increase. The increase in flow is not monotonic for H4, perhaps due to the extent of the decrease in flow through H1 and H3 as their capacities are reduced from 172 units (see Figure 6).

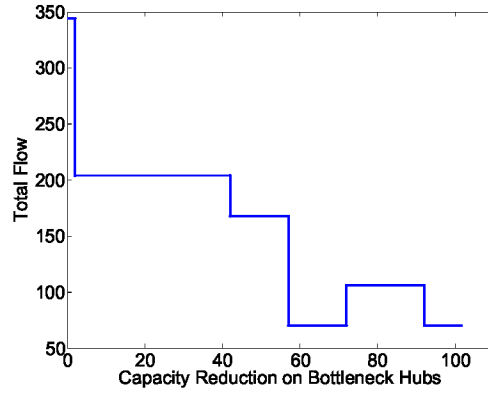
capacity on each H1 and H3 drops below 115 units, or 57 below the unconstrained level of operation at both these airports. While this provides some evidence that slot restrictions result in the greatest amount of extra work for the most-connected airport, this could also be a case of the minimally-utilized hub in the unconstrained case eventually having its excess “capacity” taken advantage of as lesser-connected hubs H2 and H4 are closer to maximum levels of operation after already beginning at higher levels of outgoing flow.

Rather than exercise a slot restriction on multiple airports at once, we now constrain just one of the two bottleneck hub airports (H1 instead of both H1 and H3) from the unconstrained optimal MINNTHRU solution and then proceed by sequentially adding to this initial constraint by imposing a slot restriction on each new bottleneck airport arising from successively more constrained optimal MINNTHRU solutions. We begin by arbitrarily selecting H1, and constrain the outgoing flow of aircraft to be no more than 167 units resulting in the optimal MINNTHRU solution shown in the second row of Table 1. The hub H2 now becomes the new bottleneck airport and the outgoing flow of 226 units becomes the new worst-case level of operation. As we proceed in increasing the slot restrictions on the hub airports, we observe in Table 1, that the minimum worst-case levels of operation over the entire network do generally increase but not monotonically so. For example, when we add a slot restriction on hub H3 to the initial restrictions on hubs H1 and H2, the worst-case level of operation decreases from 226 to 211.

Both the results in Figure 5 and Table 1 present apparent peculiarities. In Figure 5, the increase in traffic flow through the unconstrained hub airports is non-monotonic as we observe a sudden drop in the flow outgoing from H4 – while the outgoing flow from the other two unconstrained hubs remains constant – as the constraints on both H1 and H3 become more severe (in this case, the upper bound on flow for both decreases from 105 to 100). In Table 1, we see that at each step, after placing a slot restriction on the identified bottleneck hub, the flow of aircraft through that hub decreases well below the capacity on flow specified by the slot restriction. However, it seems that the topology of the network from Figure 4 contributes towards both of these results. Table 2 presents the number of spokes which are connected to each distinct pair of hubs, and demonstrates the “symmetry” that exists in the network in Figure 4. We observe that no fewer than six spoke airports can be commonly connected to each distinct pair of hubs – meaning that every commodity (a spoke-to-spoke itinerary) associated with these spokes may be routed through multiple hubs. Thus, if a particular hub is constrained, a significant number of commodities routed through that hub can and will be routed through another hub. For example if only H1 is slot-restricted, then commodities associated with up to nine spoke airports may be routed instead through H2 alone, which may help explain the substantial “slack” encountered after a bottleneck hub becomes slot-restricted. This slack also seems to play a role in the non-monotonicity we observe in the increase in the outgoing flows from unconstrained hubs H2, H4 and H5 from Figure 4. Figures 6a and 6b show, respectively, the total outgoing flows through all unconstrained and all constrained hubs as a function of increasing severity of the slot restrictions on the constrained hubs. We see from Figure 6b that the one decrease in flow through unconstrained hubs corresponds to an expected increase through constrained hubs where each H1 and H3 are constrained to 72 units of flow below their unconstrained level. However, the total decrease in flow through H1 and H3 is more than 250 units of flow below their cumulative unconstrained level, meaning a substantial amount of slack exists at both H1 and H3 at that particular level of the slot restrictions. This slack, along with the possibility of multiple optima from the nonlinearity of the minimax objective function, may contribute to the non-monotonicity seen in Figure 5.



(a) Unconstrained hubs



(b) Constrained hubs

Figure 6. Total flow on the unconstrained (H2, H4, H5) and constrained (H1,H3) hub airports in the network from Figure 3, as a function of the reduction in capacity of H1 and H3 from their unconstrained level of operation, while all spoke airports remain unconstrained. Flow through the constrained hubs decreases as capacity on them is reduced, shifting to the unconstrained hubs, although these relationships are not monotonic. The initial reduction in capacity at both H1 and H3 by two units of flow produces a substantially larger reduction in flow through these two constrained hub airports. This unused capacity at both H1 and H3 perhaps contributes to the non-monotonicity of the increase (decrease) in flow through the unconstrained (constrained) hubs.

Capacity					Outgoing Flow				
H1	H2	H3	H4	H5	H1	H2	H3	H4	H5
∞	∞	∞	∞	∞	172	138	172	84	50
167	∞	∞	∞	∞	48	226	120	172	50
167	221	∞	∞	∞	80	64	226	196	50
167	221	221	∞	∞	99	211	172	84	50
167	206	221	∞	∞	48	142	120	256	50

Table 1. Optimal flow levels for each hub airport (last five columns) and corresponding capacities for each hub (first five columns) in each row. First row corresponds to the unconstrained case, and each subsequent row corresponds to a case where a hub either becomes slot-restricted (by placing a finite capacity) or increases the severity of its slot restriction (by reducing the capacity). Bold values indicate the minimum worst-case level of operation for the given instance of the network.

	H1	H2	H3	H4	H5
H1	x	9	6	8	9
H2	x	x	6	9	12
H3	x	x	x	8	13
H4	x	x	x	x	11

Table 2. Number of spoke airports connected to each distinct pair of hubs in the network in Figure 4

6 Concluding Remarks

In response to increasing delays and congestion at airports within the national airspace system, much attention has been devoted to studying strategies for managing demand and capacity for scarce airport resources both on a short-term level and on longer time horizons for the purpose of constraining or modifying airport demand patterns. Most strategies for adjusting congestion focus on congestion management at individual airports, without fully recognizing that there are global consequences affecting the entire national airspace. In view of a need for less localized models for strategic decision-making in air transportation, this paper has presented a model for global analysis of the NAS, particularly as it operates with some degree of local congestion. The MINNTHRU model defines a high-level aggregate model of NAS operations, without specifying all details of aircraft, airline and airport operations, to provide policy makers and regulatory authorities a simple and quick tool for evaluating potential congestion and capacity issues within the system and the effects of decisions to mitigate these issues. This model specifically evaluates congestion as it arises solely from an airport network topology, with the objective of producing a flow of aircraft which minimizes the load on the busiest airport.

One type of decision for reducing congestion is the use of slot restrictions on congested airports. In a set of case studies on one small network and one medium-sized, “NAS-like” network, we show how MINNTHRU may be used to assess the effects of slot restrictions on the operation of an entire airport network. For both small and medium networks, which are hub-spoke type networks, stricter slot restrictions on a congested hub airport lead to increases in the minimum worst-case operation of any airport and the operation of all unconstrained hub airports. While these case studies demonstrate the capacity for MINNTHRU to be used in supporting decisions made by airspace users to manage congestion through the NAS, they are performed on simplified example networks without accounting for actual flight itineraries and connections as may be found using data from the BTS Origin and

Destination Survey [36]. Furthermore, in our analysis, we varied the level of slot restrictions at airports and observed the global responses. A more relevant question for policymakers and regulatory authorities is what the optimal slot restriction is. The development of a true strategic NAS testbed for evaluating this and other issues concerning the system-wide impact of slot restrictions will require a version of MINNTHRU populated with exactly this kind of data. We anticipate that the future of our model will provide this analysis, as well as a process of comparison and validation to the known results of previous slot restrictions in practice.

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14. ABSTRACT <p>This paper proposes a model for evaluating long-term measures to reduce congestion at airports in the National Airspace System (NAS). This model is constructed with the goal of assessing the global impacts of congestion management strategies, specifically slot restrictions. We develop the Minimax Node Throughput Problem (MINNTHRU), a multicommodity network flow model that provides insight into air traffic patterns when one minimizes the worst-case operation across all airports in a given network. MINNTHRU is thus formulated as a model where congestion arises from network topology. It reflects not market-driven airline objectives, but those of a regulatory authority seeking a distribution of air traffic beneficial to all airports, in response to congestion management measures. After discussing an algorithm for solving MINNTHRU for moderate-sized (30 nodes) and larger networks, we use this model to study the impacts of slot restrictions on the operation of an entire hub-spoke airport network. For both a small example network and a medium-sized network based on 30 airports in the NAS, we use MINNTHRU to demonstrate that increasing the severity of slot restrictions increases the traffic around unconstrained hub airports as well as the worst-case level of operation over all airports.</p>					
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